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Solutions to written exam for the M. Sc in Economics
Economics of Exchange Rates

August 28, 2018

Number of questions: This exam consists of 2 questions.

1. **Central Bank intervention**

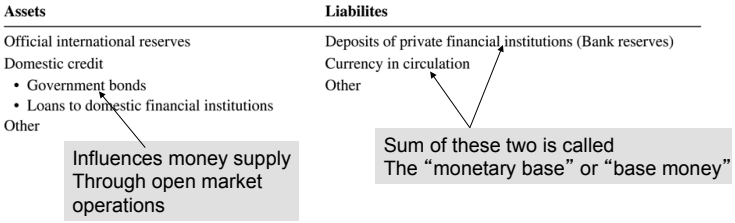
   (a) Explain the difference between a sterilized, a non-sterilized and an oral intervention.

   Answer:

   - Central bank intervention: The monetary authority (the central banks) buys or sells foreign exchange in an attempt to influence future currency movements.
   - Non–sterilized official intervention: The monetary authorities buy or sell foreign exchange such that the monetary base is affected.
   - Sterilized official intervention: The monetary authorities buy or sell foreign exchange and sterilize the effects on the monetary base, they simultaneously sell or buy domestic bonds.
   - Oral intervention: Public statements, i.e. speeches, interviews and public testimonies, about the domestic exchange rate but without any central bank intervention.

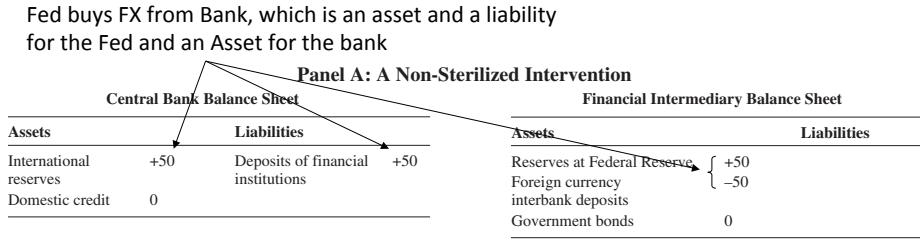
   (b) Explain using the central bank balance sheet how a sterilized and a non-sterilized intervention may be carried out.

   Answer: Consider the central bank balance sheet below

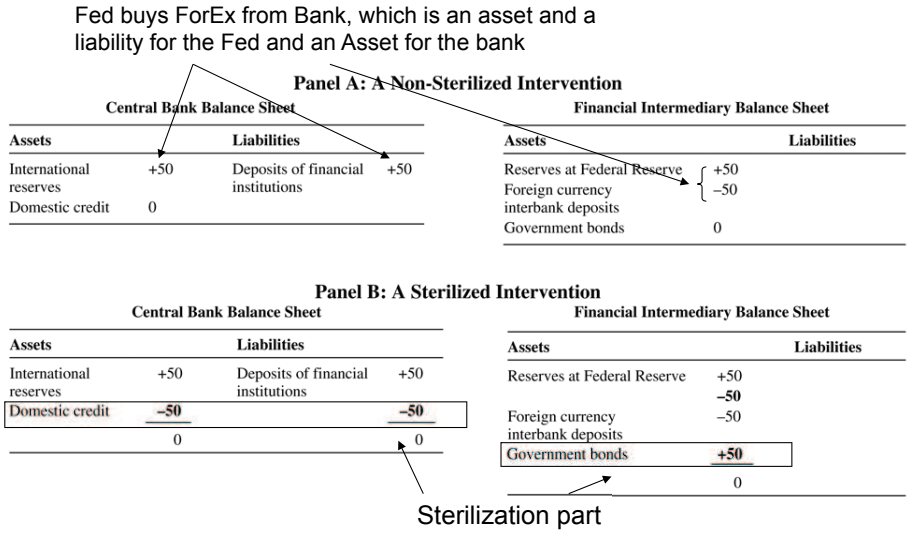


   Non-sterilized intervention: Federal Reserve wants to depreciate the dollar relative to the yen (the dollar is too strong) and suppose the exchange rate is ¥100/$. To carry out this intervention,the Fed buys ¥5,000 million in the foreign exchange market from a major U.S. commercial bank using an account at a commercial bank. The total cost will be $50 million = (¥5,000 million)/(¥100/$). In return, the commercial bank wires ¥5,000 million to the Fed. The commercial bank will be unaffected,

assets decrease by ¥5,000 million and increase by \$50 million. At the central bank, this non-sterilized intervention increases foreign assets and therefore increases the U.S. money supply. The Fed pays the bank by creating \$50 million of base money, or in other words, the monetary base increases by \$50 million. These transactions are illustrated below.



A sterilized foreign exchange intervention: In order to prevent the money supply to be affected, the central bank can combine the operation above by selling domestic bonds to the commercial bank. The effect is that domestic credit and bank's reserves at the central bank decrease and holdings of domestic bonds at the bank increase. The central bank is selling domestic bonds to the bank and buys back the cash it created during the non-sterilized intervention (the central bank buys \$50 million). Fed sells \$50 million worth of domestic government bonds to a commercial bank. The bank pays for these bonds using its reserve account at the Fed. This implies that money is taken out of circulation. The monetary base decreases by \$50 million. The two transactions (the non-sterilized and the sterilized intervention) cancel out. Net effect: Fed has replaced domestic bonds with foreign assets, but there is no effect on the money supply. The private sector now holds more domestic bonds and fewer foreign currency bonds. This is illustrated below.



(c) What is the main objective of Danish monetary policy?

Answer: The main objective of Danish monetary policy is to ensure that the krone

remains stable vis-à-vis the euro. The government conducts its fiscal policy and economic policy in general so as to achieve stable economic development and in accordance with a fixed exchange rate.

The overall objective is low inflation achieved by keeping the krone stable vis-à-vis the euro.

There is a clear distribution of responsibility for economic policy. The monetary policy of *Danmarks Nationalbank* is aimed solely at ensuring that the krone remains stable vis-à-vis the euro. The *government* conducts its fiscal policy and economic policy in general so as to achieve stable economic development and in accordance with a fixed exchange rate.

(d) How does the Danish central bank, Nationalbanken, use central bank intervention and other monetary policy instruments to carry out its objective?

Answer: Danish central bank is using sterilized interventions during stable market conditions whereas during more turbulent conditions, the bank is using non-sterilized intervention, i.e., a combination of interest rate changes and interventions.

(e) Are Danish central bank interventions effective? Put the effectiveness in an international perspective.

Answer: As in the international literature, we can reject that direction and stabilization criteria are not fulfilled. There are very few episodes when interventions are combined with interest rate changes, which is a different pattern compared to what other central banks do in practice. So, therefore the overall conclusion is that interventions have been effective.

2. **FX market micro structure**

(a) Consider the following standard two-country micro based macro model:

$$s_t = \mathbb{E}_t^D[s_{t+1}] + \hat{r}_t - r_t - \delta_t \tag{1}$$

$$\mathbb{E}_t^D(\hat{r}_{t+i} - r_{t+i}) = (1+\gamma_\pi)\mathbb{E}_t^D(\Delta\hat{p}_{t+1+i} - \Delta p_{t+1+i}) + \gamma_y \mathbb{E}_t^D(\hat{y}_{t+i} - y_{t+i}) - \gamma_\varepsilon \mathbb{E}_t^D \varepsilon_{t+i} \tag{2}$$

$$\varepsilon_t = s_t + \hat{p}_t - p_t \tag{3}$$

where notation is standard. Explain the underlying assumptions of this model and the rationale behind the equations stated above.

Answer: Main assumptions are: There are 2 two countries populated by a continuum of risk-averse agents, indexed by $n \in [0, 1]$; and D risk-averse dealers who act as market makers in the spot market for foreign currency. Two central banks, FED and ECB that both conduct monetary policy by setting short-term nominal interest rates conditional on inflation, output and real exchange rates. We abstract from interdealer trading.

Equation (1): The log spot price quoted by all dealers conditional on dealers' common information Ω_t^D at the start of week t is determined by the interest differential and

the risk premium δ_t. The equation implies that the price quoted by all dealers at the start of week t is equal to the payoff from holding foreign currency until next week $\mathbb{E}_t^D s_{t+1} + \hat{r}_t - r_t$ less a risk premium δ_t. The risk premium is determined by the requirements of efficient risk sharing, dealers will choose δ_t such that their holdings of risky currencies at the end of the week t is zero.

Equation (2): Shows that dealers' expect that the interest rate differential depends on (i) the future expected inflation differential, (ii) the future difference between output gaps, and (iii) the real exchange rate $\mathbb{E}_t^D \varepsilon_t \equiv s_t + \hat{p}_t - p_t$ which is equation (3).

(b) Show that the nominal exchange rate (under the assumption of no bubbles) can be written as

$$s_t = (\hat{r}_t - r_t) + \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i f_{t+i} - \mathbb{E}_t^D \sum_{i=0}^{\infty} \rho^i \delta_{t+i} \tag{4}$$

where

$$f_t = (1+\gamma_\pi)(\Delta\hat{p}_{t+1} - \Delta p_{t+1}) + \gamma_y (\hat{y}_t - y_t) + \tfrac{1-\rho}{\rho}(p_t - \hat{p}_t). \tag{5}$$

provide an interpretation of these equations.

Answer: In order to derive equation (4) and (5) we first use equation (1)

$$s_t = \mathbb{E}_t^D[s_{t+1}] + \hat{r}_t - r_t - \delta_t$$

for period $t+1$ taking expectations conditional on information available at time t

$$\mathbb{E}_t^D s_{t+1} = \mathbb{E}_t^D [s_{t+2} + \hat{r}_{t+1} - r_{t+1} - \delta_{t+1}]$$

Equation (2) states that

$$\mathbb{E}_t^D(\hat{r}_{t+i} - r_{t+i}) = (1+\gamma_\pi)\mathbb{E}_t^D(\Delta\hat{p}_{t+1+i} - \Delta p_{t+1+i}) + \gamma_y \mathbb{E}_t^D(\hat{y}_{t+i} - y_{t+i}) - \gamma_\varepsilon \mathbb{E}_t^D \varepsilon_{t+i}$$

Define the fundamentals as equation (5). Solve the equation following from equation (1) above and replace interest differential with equation (2). This yields

$$\mathbb{E}_t^D s_{t+1} = \mathbb{E}_t^D [(1+\gamma_\pi)(\Delta\hat{p}_{t+2} - \Delta p_{t+2}) + \gamma_y (\hat{y}_{t+1} - y_{t+1}) + \gamma_\varepsilon \varepsilon_{t+1} - \rho_{t+1} + s_{t+2}]$$

and insert the expression for the real exchange rate (equation(3))

$$\mathbb{E}_t^D s_{t+1} = \mathbb{E}_t^D [(1+\gamma_\pi)(\Delta\hat{p}_{t+2} - \Delta p_{t+2}) + \gamma_y (\hat{y}_{t+1} - y_{t+1}) + \gamma_\varepsilon (s_{t+1} + \hat{p}_{t+1} - p_{t+1}) - \rho_{t+1} + s_{t+2}]$$

simplify (divide by $(1+\gamma_\varepsilon)$) such that

$$\mathbb{E}_t^D s_{t+1} = \mathbb{E}_t^D [\rho((1+\gamma_{pi})(\Delta\hat{p}_{t+2} - \Delta p_{t+2}) + \gamma_y (\hat{y}_{t+1} - y_{t+1}) - (1-\rho)(\hat{p}_{t+1} - p_{t+1})) -$$

$$\rho\delta_{t+1} + \rho s_{t+2}]$$

where $\rho = \frac{1}{1+\gamma_\varepsilon}$. Use definition of fundamentals in equation (5) such that

$$\mathbb{E}_t^D s_{t+1} = \mathbb{E}_t^D [\rho f_{t+1} - \rho\delta_{t+1} + \rho s_{t+2}]$$

Solve this difference equation

$$\mathbb{E}_t^D s_{t+1} = \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i \left(f_{t+i} - \delta_{t+i}\right) + \mathbb{E}_t^D \lim_{i\to\infty} \rho^i s_{t+1+i}$$

Under the assumption of no bubbles, the last term on the right hand side converges to zero. Finally, from equation (1) we have that

$$\mathbb{E}_t^D s_{t+1} = s_t - (\hat{r}_t - r_t - \delta_t)$$

implying that

$$s_t = (\hat{r}_t - r_t) + \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i f_{t+i} - \mathbb{E}_t^D \sum_{i=0}^{\infty} \rho^i \delta_{t+i}$$

This equation implies that the spot rate is determined by (i) the current stance of monetary policy in the two countries (the interest differential affects dealers' quotes), (ii) the future course of macro fundamentals (dealers' expectations of the central bank response to macroeconomic conditions), and (iii) risk-sharing between dealers and agents (present and future expected values of the risk premium).

(c) If we assume that aggregate demand for foreign currency is given by

$$\alpha_t = \int_0^1 \alpha_t^n \, dn = \alpha_s(\overline{\mathbb{E}}_t^n s_{t+1} - s_t + \hat{r}_t - r_t) + h_t \tag{6}$$

and if we also invoke the risk sharing condition $\mathbb{E}_t^D \alpha_t = 0$, we find that the risk premium can be written as

$$\delta_t = \mathbb{E}_t^D \left[s_{t+1}^e - \tfrac{1}{\alpha_s} h_t \right] \tag{7}$$

where $s_{t+1}^e = s_{t+1} - \overline{\mathbb{E}}_t^n s_{t+1}$. Discuss the implications of the risk premium relation. How are order flows linked to the risk premium?

Answer: Equation (7) implies that the risk premium depends on (i) aggregate hedging demand $\mathbb{E}_t^D h_t$, and (ii) the average error agents make when forecasting next week's spot rate s_{t+1}^e. Two implications can be drawn: Dealers lower the risk premium when they anticipate a rise in the aggregate hedging demand for euros, the implied fall in the excess return agents expect offset their desire to accumulate larger euro holdings. Dealers lower the risk premium to offset agents' desire to accumulate larger euro holdings when they are viewed as too optimistic about the future spot rate, i.e., when $\mathbb{E}_t^D s_{t+1} < \mathbb{E}_t^D \overline{\mathbb{E}}_t^n s_{t+1}$.

(d) Show that equation (4) can be written as

$$s_t = (\hat{r}_t - r_t) + \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i f_{t+i} + \tfrac{1}{\alpha_s} \mathbb{E}_t^D \sum_{i=0}^{\infty} \rho^i h_{t+i} - \tfrac{1}{\rho} \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i s_{t+i}^e \tag{8}$$

Interpret this relation and discuss whether this model can solve the disconnect puzzle?

Answer: To derive equation (8) we use the result in equation (4) and the definition of the risk premium in equation (7). Insert (7) into (4) and simplify to obtain

$$s_t = (\hat{r}_t - r_t) + \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i f_{t+i} + \tfrac{1}{\alpha_s} \mathbb{E}_t^D \sum_{i=0}^{\infty} \rho^i h_{t+i} - \tfrac{1}{\rho} \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i s^e_{t+i}$$

Interpretation: First note that the model is different from standard macro models, s_t is a function of information available to dealers Ω_t^D, at the time they quote prices. This information set includes contemporaneous interest rates but not other contemporaneous fundamentals. The risk premium incorporates dealers' estimates of aggregate hedging demand and agents' forecast errors. These factors can be a source of variation in the spot rate if dealers view the current and future monetary policy unchanged. Standard models suggest that spot rates depend on current and expected future fundamentals. But in this model, dealers expectation of the agents' average forecast errors affect the spot rate via their implication for risk-sharing.

Exchange-rate disconnect puzzle: Macro models cannot predict the behavior of spot exchange rates at frequencies higher than a year. The model above suggests that order flows are important which is consistent with empirical evidence. The model above also suggests that order flows can reveal information about the future state of the economy, a hypothesis not rejected by empirical evidence. In this sense, the model contributes to an explanation of the disconnect puzzle.